# Influence of Crack Parameters and Loading Direction on Buckling Behavior of Cracked Plates Under Compression

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**Abstract**

The aim of the present paper is to investigate buckling phenomenon of various cracked plates under compression load. The finite element procedure (ANSYS Package) is used to determine the critical buckling load by considering the effects of crack length and crack location (i.e. crack parameters) as well as loading direction parallel or perpendicular with respect to crack faces. It is found from the obtained results which are summarized graphically in figures that the crack parameters and loading direction have significant effects on the critical buckling load (i.e. increased or decreased) of compressed cracked plates. The effects of these factors are discussed in detail. The useful and interesting conclusions drawn from this work will be helpful for health monitoring or condition assessment of aging plated structures with cracking damages.

## تأثير محددات الشق و أتجاه التحميل على تصرف الانبعاج للصفائح المتشققة تحت الانضغاط

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الملخص :

الهدف من البحث هو دراسة ظاهرة الانبعاج للصفائح مختلفة متشققة تحت حمل الانضغاط. أستخدمت طريقة العناصر المحددة (برنامج ANSYS) للحساب حمل الانبعاج الحرج بأخذ بنظر الاعتبار تأثير طول الشق و موقع الشق (محددات الشق) بالإضافة إلى اتجاه التحميل موازي أو عامودي بالنسبة إلى وجهي الشق. وجد من النتائج المستحصلة و التي لخصت تخطيطاً بأشكال بأن محددات الشق و اتجاه التحميل لهما تأثير واضح على حمل الانبعاج الحرج (يقل أو يزيد) للصفائح المتشققة المضغوطة. تأثيرات هذه العوامل نوقشت بالتفصيل. الاستنتاجات المفيدة و المهمة التي تم الحصول عليها من هذا العمل ستكون مساعدة لمراقبة صحة أو شرط تقييم التراكيب المتكونه من الصفائح الهرمة مع أضرار التشقق.

## 1. Introduction

The investigation and understanding of both behavior and strength of plates in intact , defected and damaged is very important because majority of thin walled structures are widely used plates. Dealing with the fields of mechanical, civil, aerospace and marine engineering (e.g. plate girders, automotive aircraft and submarine). These structures are prone to suffer various types of damages and defects such as initiation and propagation of cracks, corrosion and chemical attack, as they get older under variety of load combinations. Presence of these imperfections may severely compromise the load carrying capacity and their structural safety assessment.

According to the literature review there are numerous researches have been worked out on cracked plates for different situations. Buckling phenomenon (i.e. critical buckling load and buckling coefficient) and their relationship with crack parameters such as crack length, crack orientation and crack location as well as plate strength are the focus of researches under compression, tension and shear loads using different types of solution techniques. Some studies have been carried out to evaluate the influence of cracks on the buckling load in plates or composite plates under tension [1-5]; in such cases buckling it usually appears as a local phenomenon in compressed regions around the cracks. The behavior of cracked plates and shells under internal pressure and axial compression or tension has been the subject of Vafai and Estekanchi investigations [6-9]. They showed that a careful mesh refinement at the crack tip would give reliable results and also they computed the free vibration frequencies and the corresponding mode shapes of these plates. Kumar and Paik [10] used the hierarchical trigonometric functions to define the displacement function of cracked plate and then to estimate buckling loads under uniaxial and biaxial compressive load as well as in-plane shear load. Their results are found to correlate well with those obtained using a finite element method. Brighenti [11,12] investigated the effect of crack length, crack locations, Possion's ratio and boundary conditions (supported or clamped edges) on the critical buckling load of cracked rectangular plates under tension or compression. His studies have been shown that the effects of cracks on buckling phenomenon under compressive stress depend on the plates boundary conditions, while they are almost independent for tension cases. Also, he indicated the variation of the Possion's ratio over the range considered has significant effects for cracked plates under tension.

Alinia et al. [13,14] studied the influence of cracks on the residual strength and stiffness degradation of shear panels. They concluded the mesh density at crack tips plays a dominant role in the accuracy of analysis in an exceptional manner , but the regions of crack sides may have mesh refinements similar to uncracked panels. It was also shown form their study that long cracks could have significant influence on the buckling capacity of shear panels, especially if the cracks enter the tension fields. The behavior and ultimate strength of cracked plates under compression or tension experimentally and numerically were investigated extensively by Paik et al. [15-17]. They developed theoretical models for predicting the ultimate strength of plate elements with fatigue cracking [15] and employed these models for more crack configurations [17]. Recently, Khedmati et al. [18] calculated the buckling coefficients of cracked plates by varying some crack parameters and plate aspect ratio. Their investigation has shown that the behavior of the cracked plate are entirely different when the crack is either on the edge or inside the plate.

From above review it can be concluded that most of these studies were carried out on cracked plate with varying crack length or crack orientation for central or edge crack. In these studies a crack is located in the direction parallel or perpendicular to the loading direction. Few studies have been performed to evaluate the influence of crack location on the buckling behavior and ultimate strength [15-18] , but by altering crack location horizontally or vertically for central or edge crack only. Consequently in present paper, finite element analysis are performed to investigate buckling behavior of compressed cracked square plates by taking into consideration cracks at any location in a plate and loading direction parallel and perpendicular with respect to crack faces for different crack lengths. In this regard, a set of eigenvalue buckling analysis is conducted for a various cracked plates under compression. The computations are carried out using ANSYS Software version11, a commercial finite element package.

## 2- Theoretical Analysis

The governing fourth-order partial differential equation of the linear buckling analysis of plates is obtained from Von Karman plate theory [19]:

$$\nabla^4 w(x,y) = \frac{1}{D}\left( N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right) \quad ....(1)$$

Equation (1) describes the plate's deflection w(x,y) when second-order geometrical effects are taken into account (i.e. membrane loadings are considered).

Where

$D = \frac{Et^3}{12(1-\nu^2)}$ is the plate's flexural rigidity

E and ν are the Young's modulus and Poisson's ratio of the material respectively and t is the plate thickness.

$N_x$ , $N_y$ and $N_{xy}$ are the membrane and shearing forces (per unit length of the plate) in the corresponding directions.

For a simply supported plate subjected to a uniformly distributed compressive edge load acting in the x-direction only (i.e.$N_y = N_{xy} = 0$ ), as shown in figure (1), equation (1) becomes :

$$D\nabla^4 w(x,y) + N_x \frac{\partial^2 w}{\partial x^2} = 0 \quad ....(2)$$

The solution of equation (2), obtained by some analytical or numerical methods, such that introduced in reference [19] for more detail and after satisfying the simply supported boundary conditions, the critical buckling load ($N_{x,cr}$) can be expressed as:

$$N_{x,cr} = \mathrm{k}\frac{\pi^2 D}{b^2} \quad ....(3)$$

Where k is the buckling load parameter or sometime named buckling coefficient and depends only on the ratio a/b , called the aspect ratio of the plate. The variation of the buckling coefficient k as a function of the aspect ratio is found in reference [19] for other loading and boundary conditions.

The corresponding critical buckling stress is found to be :

$$\sigma_{x,cr} = \mathrm{k}\frac{\pi^2 D}{b^2 t} \quad ....(4)$$

For uncracked square plate ( c = b ) the corresponding value of the buckling coefficient is k=4 [19].

In the cases of cracked plates firstly the eigenvalue buckling analysis is performed then the relevant values of $k_{cracked}$ are determined in each case. For such cases, the following equation can be considered in order to determine the buckling strength [18]:

$$\sigma_{cr,cracked} = \mathrm{k}_{cracked}\frac{\pi^2 D}{b^2 t} \quad ....(5)$$

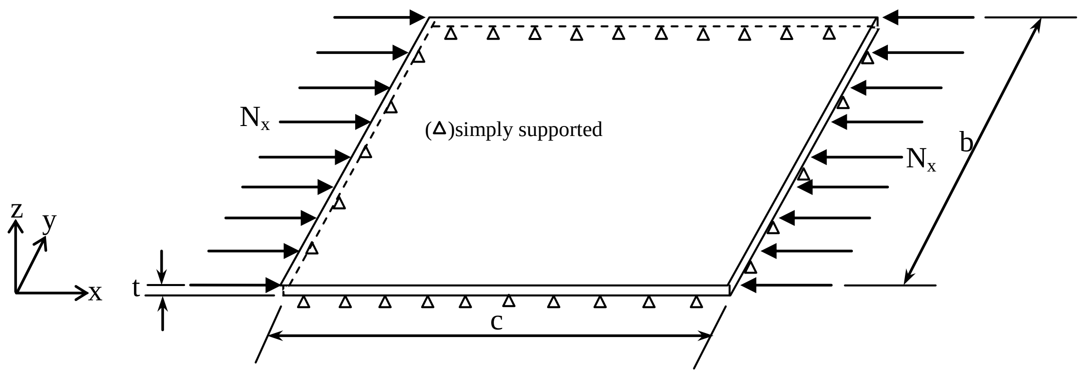



Figure (1) Simply Supported Plate Under compressive Load in x-Direction

# 3. Generation of The Finite Element Model

## 3.1 Model Description

The main problem's geometrical parameters are the plate width equal to plate length (square plate): b = c =1m and plate thickness : t =10mm. The plate material considered is supposed to be linear elastic and isotropic with Young's modulus : $E=200GN/m^2$ and Possion's ratio : ν = 0.3.

In the present study , a crack is located in the direction perpendicular or parallel to the loading direction. So, two loading cases have been investigated, which are named loading case(1) and loading case(2) when the load perpendicular or parallel with respect to the crack faces respectively. Where, the length of the crack is denoted by a, the coordinates of the crack centre are ( xo, yo ) in the x- and y-direction respectively, as shown in figure (2). Different crack lengths and crack locations were considered. In order to relate the length of cracks to the dimensions of plate, the relative crack length is defined as the ratio of the actual crack length to the plate's width. The different relative crack lengths were (a/b = 0.1 , 0.2 , 0.3 , 0.4 , 0.5 ). As well as the relative crack location is defined as the ratio of the crack coordinates (xo,yo) to the plate's width were ( xo/b = 0.1 , 0.2 , 0.3 , 0.4 , 0.5 and yo/b = 0.05 , 0.1 , 0.15 , 0.2 , 0.25 ,0.3 , 0.35, 0.4 , 0.45, 0.5 ), as indicated in table (1). In this table, shadowy bold values of yo/b corresponds to the edge cracked plate. Also when xo/b = yo/b =0.5 this represents central cracked plate.

The crack was presumed to be through thickness since thin plate is used and having no friction between their edges and no propagation was allowed. While the support members or stiffeners are located along edges, it is considered that the plate is simply supported at all four edges which are kept straight.

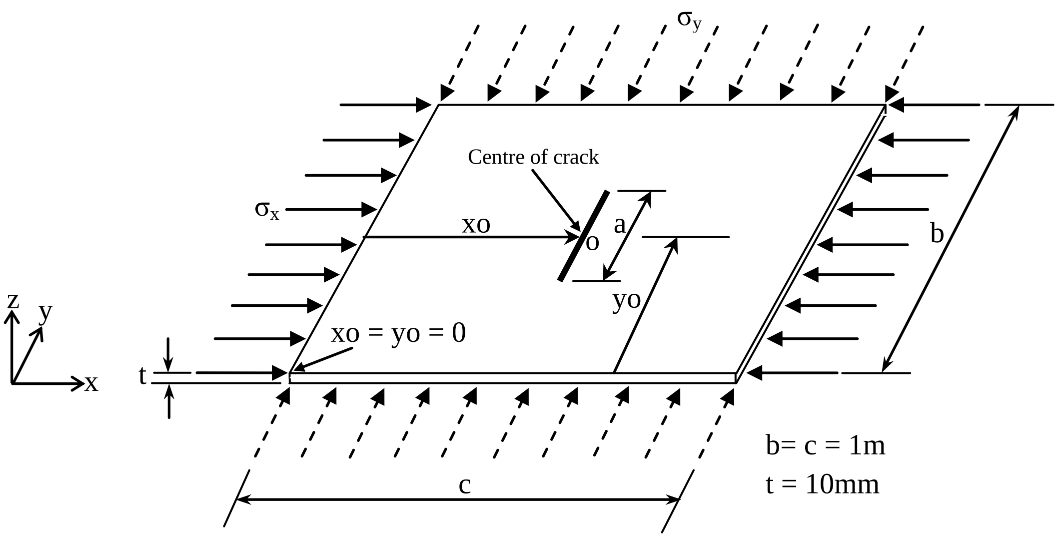



Figure (2) Cracked Square Plate with Loading Condition and Crack Location Details

Table (1) The Values of Crack Parameters in The Finite Element Model

| a/b | xo/b | yo/b |
|---|---|---|
| 0.1 | 0.1 , 0.2 , 0.3 , 0.4 , 0.5 | **0.05** , 0.1 , 0.15 , 0.2 , 0.25 ,0.3 , 0.35, 0.4 , 0.45, 0.5 |
| 0.2 | 0.1 , 0.2 , 0.3 , 0.4 , 0.5 | **0.1** , 0.15 , 0.2 , 0.25 ,0.3 , 0.35, 0.4 , 0.45, 0.5 |
| 0.3 | 0.1 , 0.2 , 0.3 , 0.4 , 0.5 | **0.15** , 0.2 , 0.25 ,0.3 , 0.35, 0.4 , 0.45, 0.5 |
| 0.4 | 0.1 , 0.2 , 0.3 , 0.4 , 0.5 | **0.2** , 0.25 ,0.3 , 0.35, 0.4 , 0.45, 0.5 |
| 0.5 | 0.1 , 0.2 , 0.3 , 0.4 , 0.5 | **0.25** ,0.3 , 0.35, 0.4 , 0.45, 0.5 |

The commercial finite element package ANSYS Software version11 is used for the modeling and buckling analysis of cracked plates. The "shell93" element of ANSYS was used for meshing procedure. This element is suitable for analysis thin-walled structures. It is a eight-node element with six degree of freedom at each node: translations in the nodal x, y, and z directions and rotations about the x, y, and z-axes. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. Figure (3) shows a sample of the finite element model for a cracked plate with crack-tip mesh refinement detail. One of the effective methods applied in practice, in order to increase the accuracy of the stress and deflection predictions is to refine the finite element mesh in the high stress area such as crack-tip region [6,7,13]. The degree of mesh refinement can be adjusted based on the required accuracy and sensitivity of the problem [7]. So, a special attention must be applied at the crack-tip region, since it has singular stress field. Around the crack-tip, finer meshes are allocated so as to provide for earlier yielding due to stress concentration [15]. More details of crack zone meshing schemes which are considered in the present study might be found in the previous studies [6,7,13].

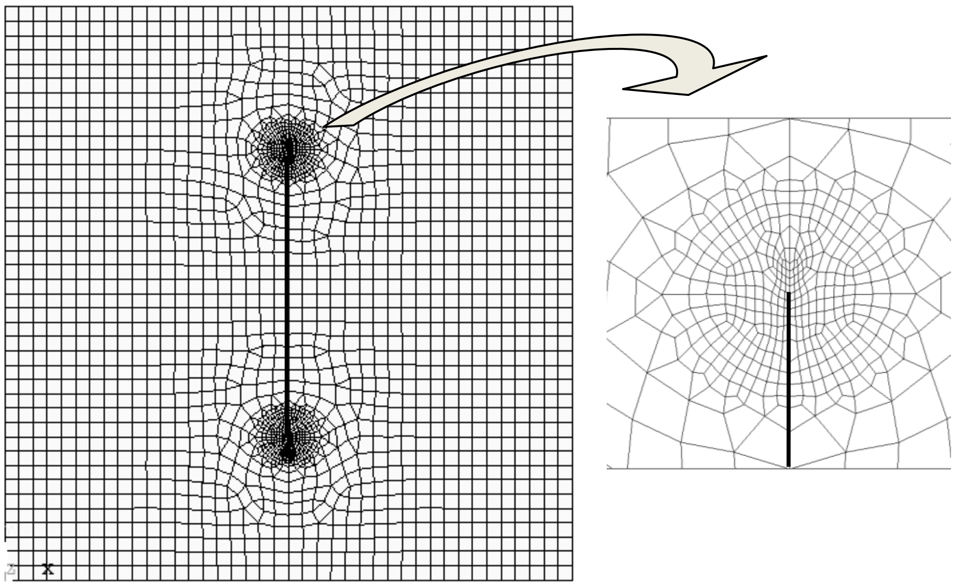
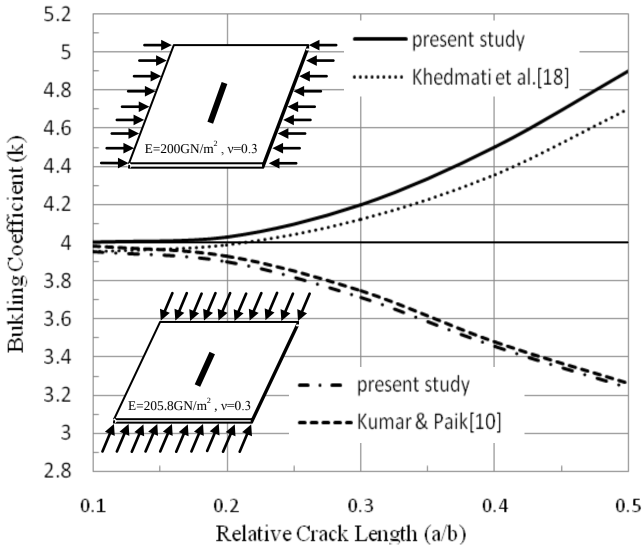


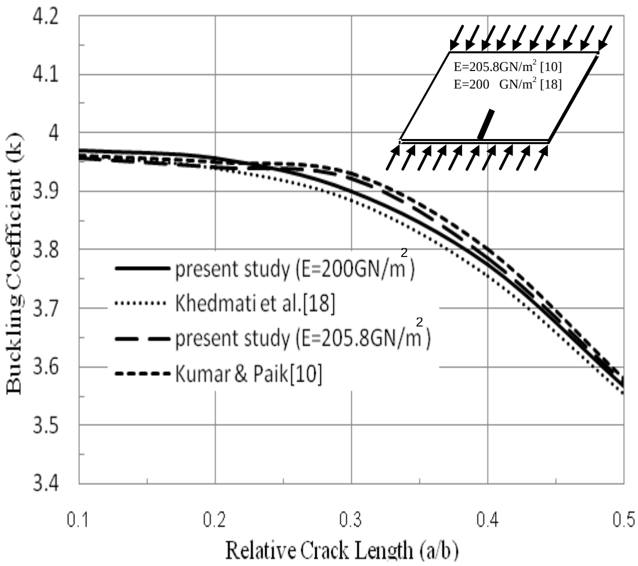
Figure (3) A Sample of The Finite Element Model For a Cracked Plate With Crack-tip Mesh Refinement Detail

## 3.2 Verification Case Studies

For verification purpose, two case studies have been selected. Edge and central cracked square plates under compression load with different relative crack length (a/b=0.1 – 0.5) which are reported by Khedmati et al.[18] and Kumar and Paik [10]. The first case study is central cracked square plate under compression load perpendicular or parallel to the crack faces as shown in figure (4a). While the second case study is edge cracked square plate under compression load parallel to the crack faces, figure (4b). This figure indicates the effect of the crack relative lengths on the buckling coefficient (k). It is clearly observed there is a good convergence between the results of the present study and those obtained by Khedmati et al.[18] and Kumar and Paik [10].




(a)Case Study(1): Central Cracked Plate




(b)Case Study(2): Edge Cracked Plate

Figure (4) Verification Case Studies

**4. Results and Discussion**

The calculated critical buckling load of cracked plates is normalized by the theoretical critical buckling load of the uncracked plate with similar geometry and material properties subject to uniform compression for first buckling mode :

$$\gamma = \frac{N_{cracked}}{N_{uncracked}}$$

Where

γ : normalized buckling load.

$N_{cracked}$ : buckling load of cracked plate obtained from finite element analysis.

$N_{uncracked}$ : buckling load of uncracked plate obtained from equation (3).

The resultants of the normalized buckling loads (γ) are displayed against the relative crack length (a/b = 0.1,0.2,0.3,0.4,0.5) for different relative crack locations in x-direction (xo/b = 0.1,0.2,0.3,0.4,0.5) and due to existing similarities in the behavior, only the case of crack locations in y-direction (yo/b = 0.5,0.45,0.4,0.35) is considered, under loading case(1) and loading case(2) in figures (5) and (6) respectively. It can be noted form figure(5) that for a given relative crack location and by making the crack approach the plate centre, the normalized buckling load increases on increasing the relative crack length by about ( 1- 1.225 ) and has highest value when xo/b = yo/b = 0.5 under loading case(1). In other words, by increasing the crack length, the associated buckling load (i.e. buckling strength) increases in comparison with an uncracked case. This can be attributed to the redistribution and turbulence of stress flow with presence of cracks, which leads to delay the occurrence of buckling in the plates, figure(7a). Approaching the crack to the loaded edge of the plate and shortening the distance between the crack and this edge, makes the stress flow would have less opportunity for redistribution and uniformity, figure(7b,7c). As a result, the whole cracked plate does not resist against the in-plane compression, consequently reduction in normalized buckling load[18]. It is interesting to note that, the presence of the crack seems to have a beneficial effect since it increase the buckling strength with respect to the uncracked case [11,12,18].

While, a lower normalized buckling load is produced by loading case(2), figure(6). This mean, by increasing the crack length, the associated buckling load decreases in comparison with uncracked case. Also, from the observation of figure(6a-d), it can be deduced that the lowest normalized buckling load corresponds to the case xo/b = yo/b = 0.5 at the plate centre. On the other hand, with approaching the crack to the supported part of the plate in its edge, the normalized buckling load increases. This behavior can be explained that when the crack is considered in the middle of the plate and parallel to loading direction, it seems that the effective length of the plate is halved and as a result the buckling load is decreased. This suggests that the increase in buckling load for plate has cracks locate in the neighborhood of the parallel edges is due to the reduction in the distance between the crack and that edges[18]. In general, the presence of a crack in a plate parallel to the loading direction makes stress flow approximately

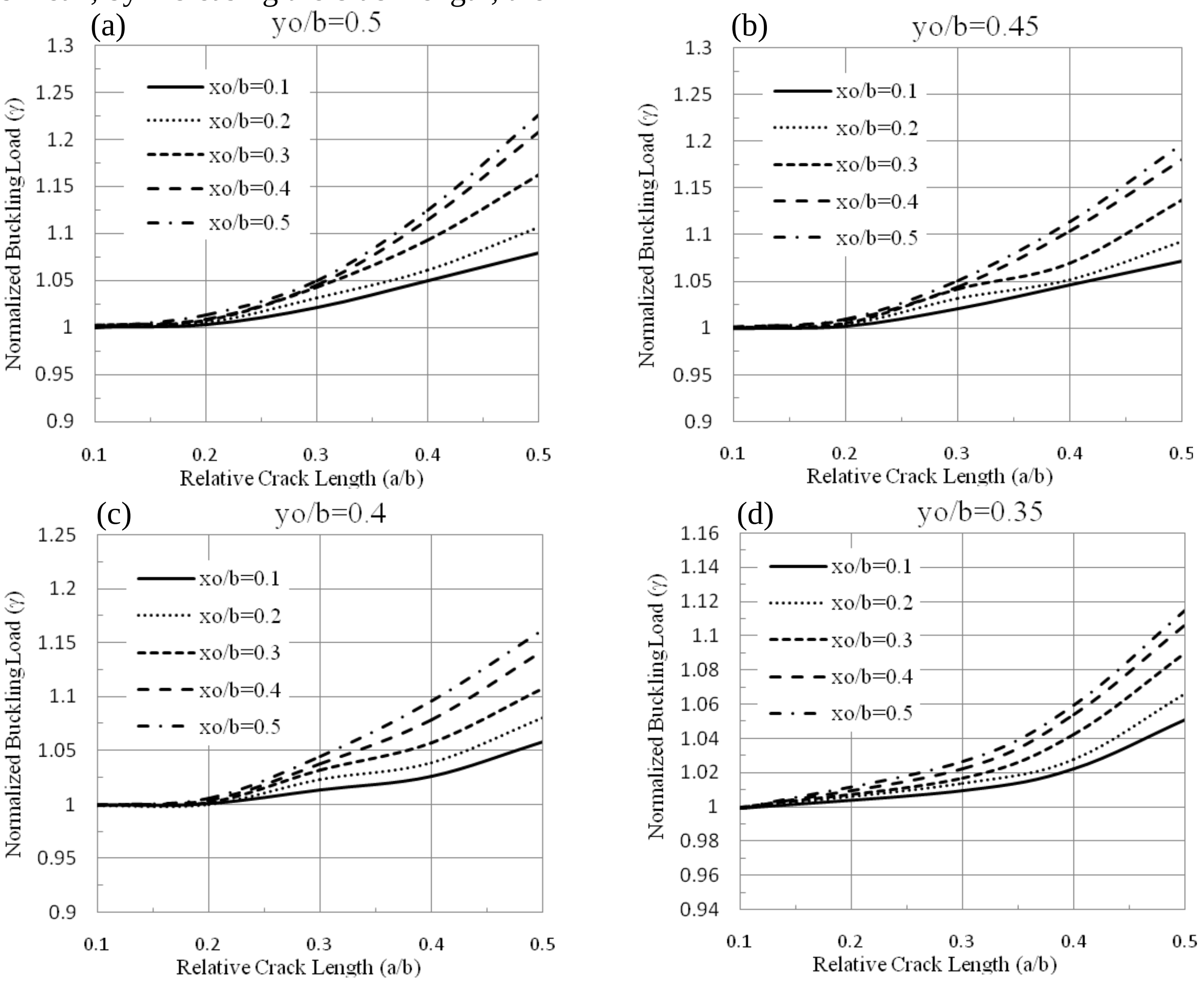



Figure (5) Variation of Normalized Buckling Load(γ) With Relative crack Length(a/b) For Different Relative Crack Locations Under Loading Case(1) uniform and straight, figure(7d-f) resulting reduction in buckling load. So, in such cases the crack can be considered to be of "damaging type" on the buckling behavior, giving rise to lower buckling stresses with respect to the corresponding uncracked ones[11,12].

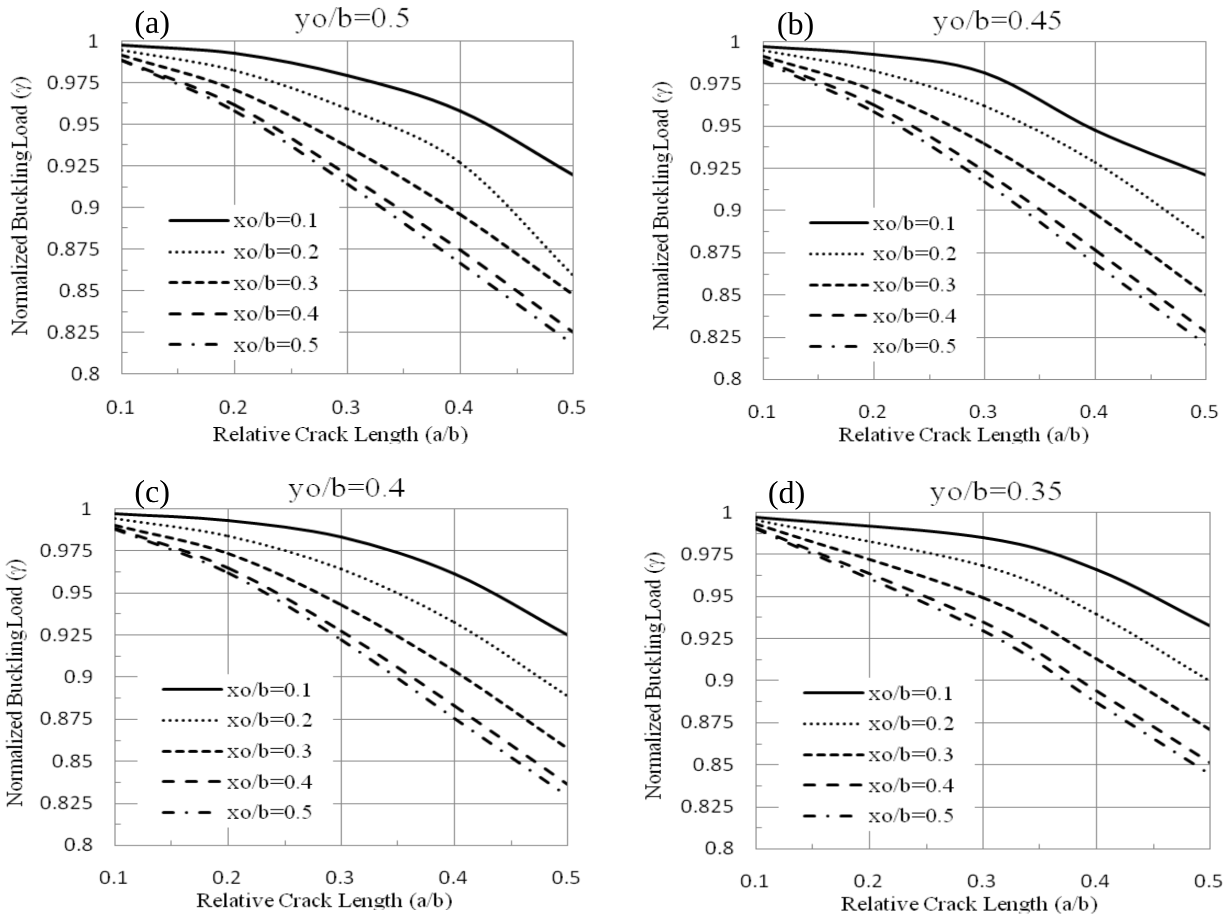



Figure (6) Variation of Normalized Buckling Load(γ) With Relative crack Length(a/b) For Different Relative Crack Locations Under Loading Case(2)

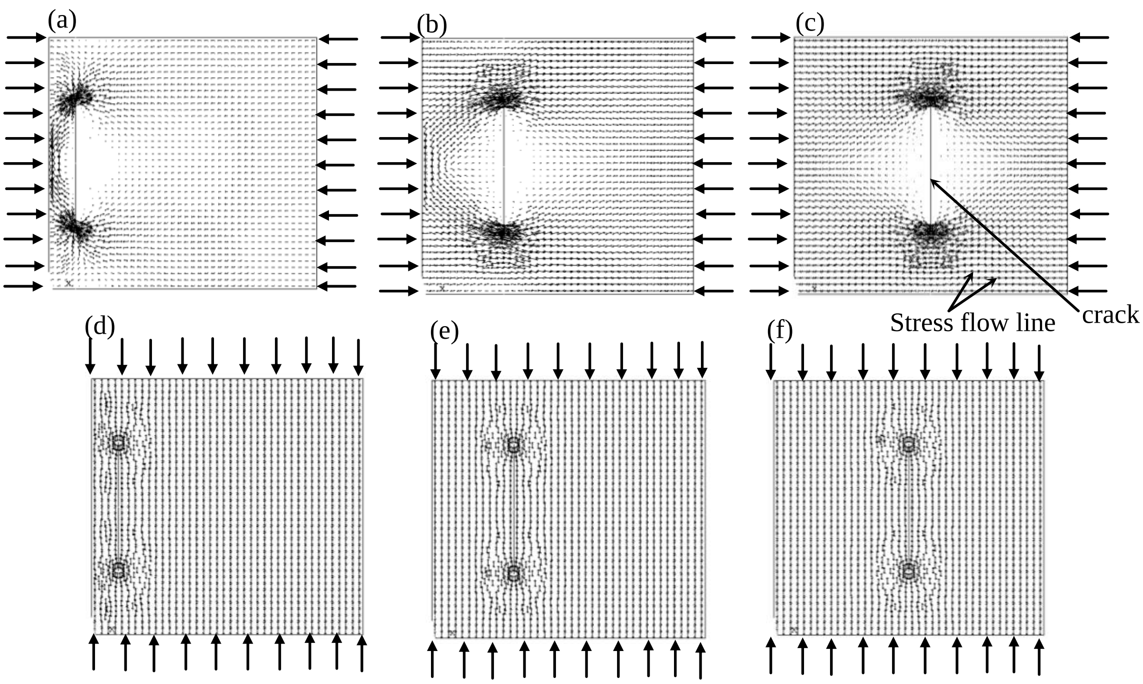



Figure (7) Redistribution of Stress Flow of a Cracked Plate With Different Crack Locations: (a,b and c)Loading Case(1) , (d,e and f)Loading Case(2)

From aforementioned discussion this explain why the buckling loads of plates with an edge crack are all below that of a prefect plate. Moreover, this is mainly due to the fact that any increase in the length of such a crack would lead to greater length of tearing of the plate from its edge, especially under loading case(1). That is why, the plate length is practically halved and as a result its buckling resistance is diminished. The larger crack length, the longer tearing of the plate[18]. Figure(8) shows the normalized buckling loads of edge cracked plate with relative crack length for different relative crack location. It is clear from this figure that the case of an edge crack perpendicular to the loading direction corresponds to the lowest buckling load ( $\gamma$ = 0.39), so can be considered to be the most dangerous one.

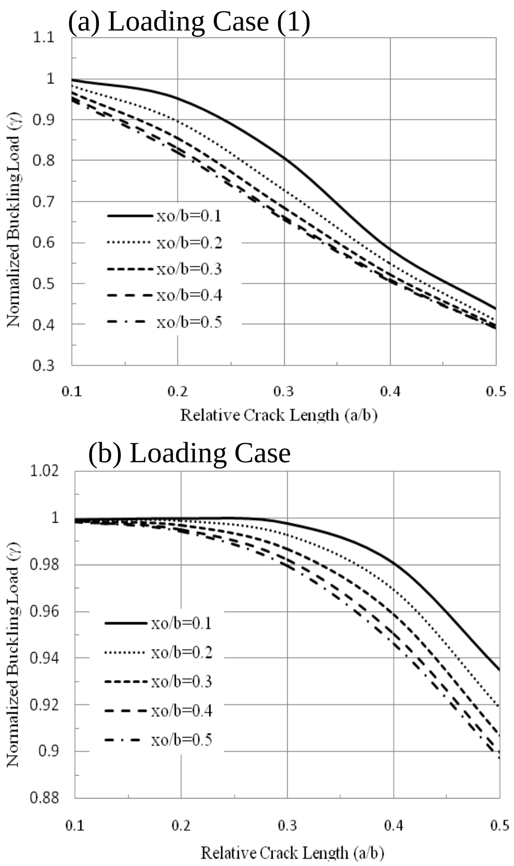



Figure (8) Variation of Normalized Buckling Load($\gamma$) With Relative crack Length(a/b) For Different Relative Crack Locations of Edge Crack Plate

Variation in normalized buckling load with relative crack location in x-direction have been plotted in figures (9) and (10) for different relative crack lengths and relative crack locations in y-direction (i.e. yo/b=0.5,0.45,0.4,0.35) under both loading situations. As can be seen in these figures, when the relative crack length is small (i.e. $0.1 \leq a/b < 0.2$), the normalized buckling load is nearly the same for all values of relative crack locations in x-direction. Moreover, a slight change in the normalized buckling load with relative crack length equal to 0.2, particularly for cracked plate under loading case(1). It is also observed that as the relative crack length increases (i.e. $a/b \geq 0.3$), the normalized buckling load is changed (i.e. increased or decreased) significantly.

In order to investigate the effect of the crack locations in y-direction on the buckling behavior, the obtained results is reported in figures(11,12) from another viewpoint for various relative crack locations in x-direction and relative crack lengths. The observation of these figures confirm that the normalized buckling load is altered lightly for the cracks at locations in y-direction yo/b = 0.5 or 0.45. As well as, it is magnified variation with the decrease in the relative crack location in y-direction (i.e. approaching the crack to the plate edge. An intermediate behavior is observed for yo/b=0.3, then the normalized buckling loading decreases when yo/b<0.3 under loading case(1),figure(11). Since the cracks approaching the plate edge. It is evident for edge cracked plate that the buckling behavior under both loading situations is almost in similar trends, decreasing in the normalized buckling load as the crack approaching plate centre, figure(13).

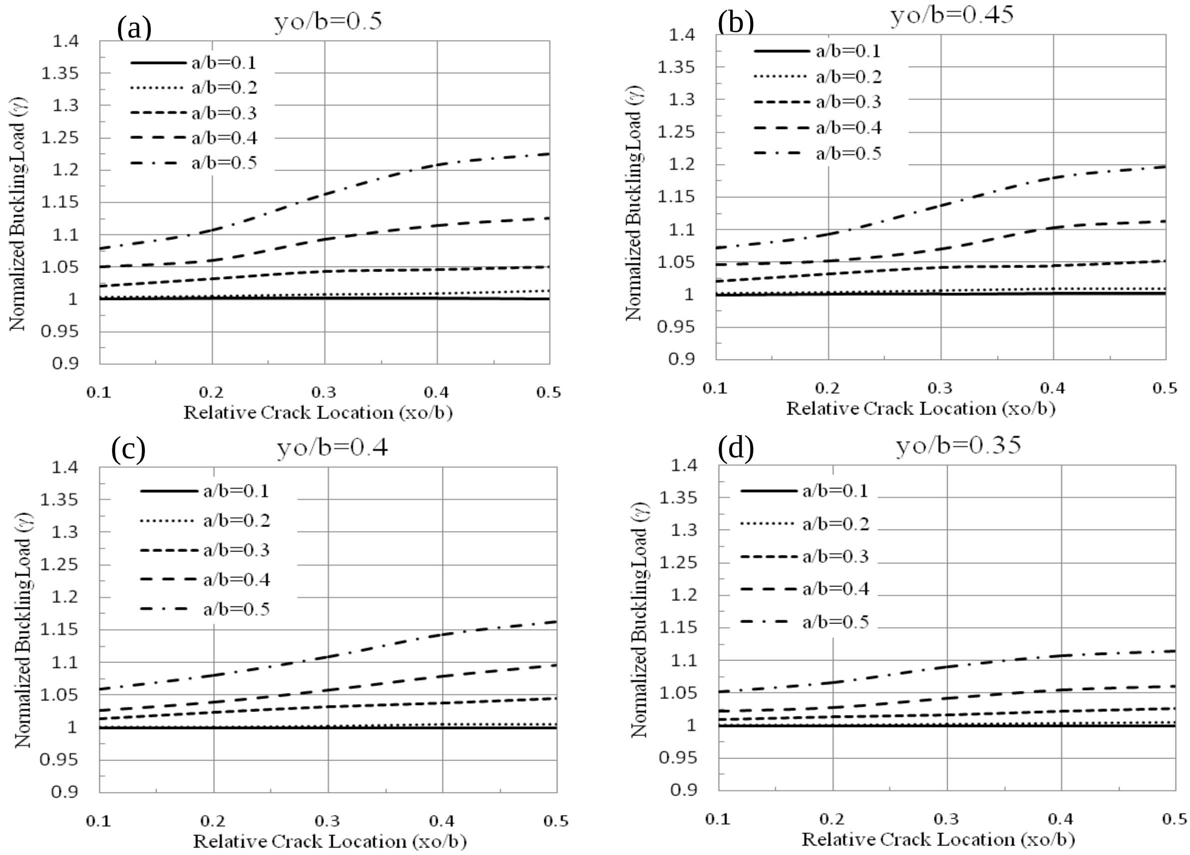



Figure (9) Variation of Normalized Buckling Load(γ) With Relative crack Location (xo/b) For Different Relative Crack Lengths Under Loading Case(1)

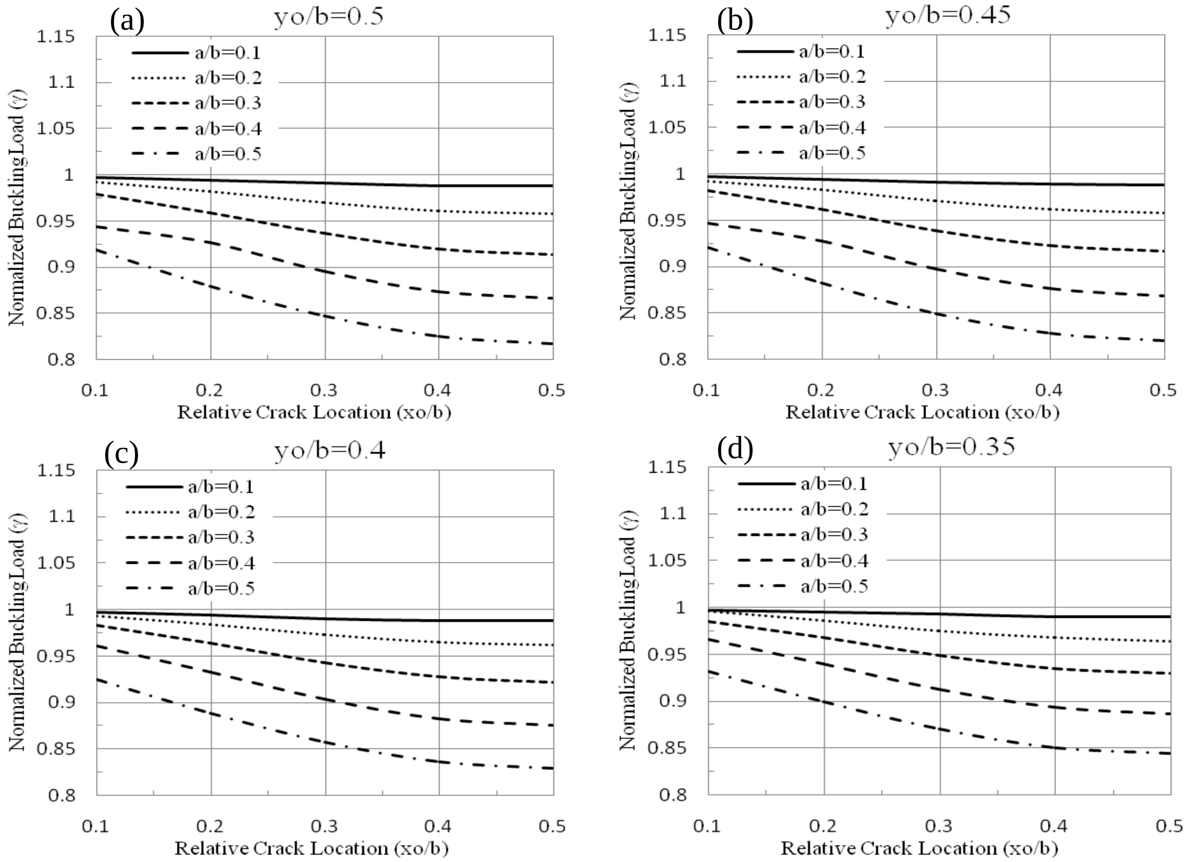



Figure (10) Variation of Normalized Buckling Load(γ) With Relative crack Location (xo/b) For Different Relative Crack Lengths Under Loading Case(2)

Finally, figure(14) shows the first buckling mode shapes only for cracked plates characterized by a/b=0.5 , xo/b=0.3 or 0.1 and yo/b=0.5 under both loading situations, since similar results (i.e. similar behavior) were obtained for cracked plates having another crack parameters for brevity. This figure is presented for graphical demonstration of the findings discussed in this work. It is observed that, the crack gap is apparently different depending on the crack parameters and loading direction with respect to the crack faces.

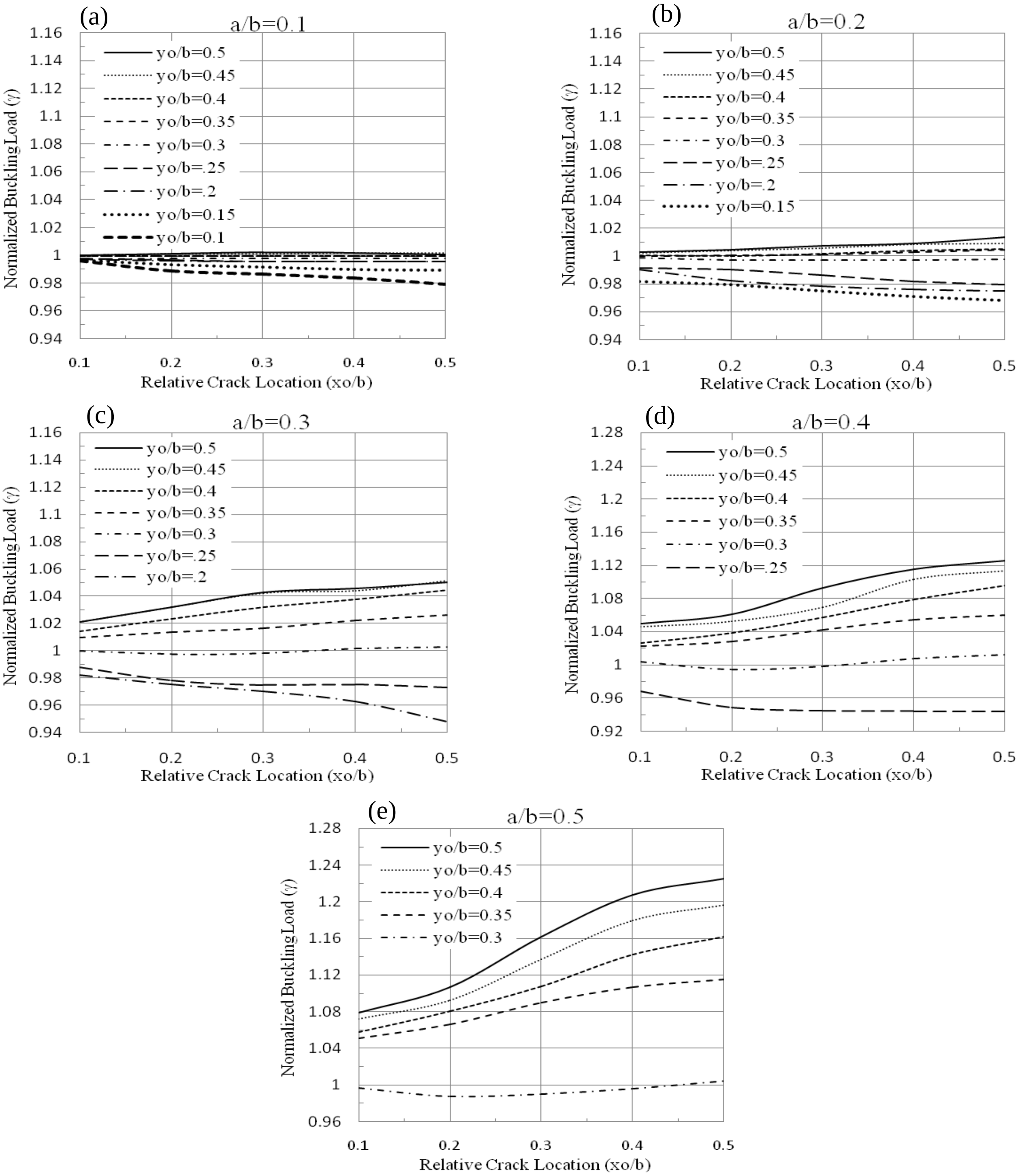



Figure (11) Variation of Normalized Buckling Load(γ) With Relative crack Location in x-direction(xo/b) For Different Relative Crack Locations in y-direction(yo/b) Under Loading Case(1)

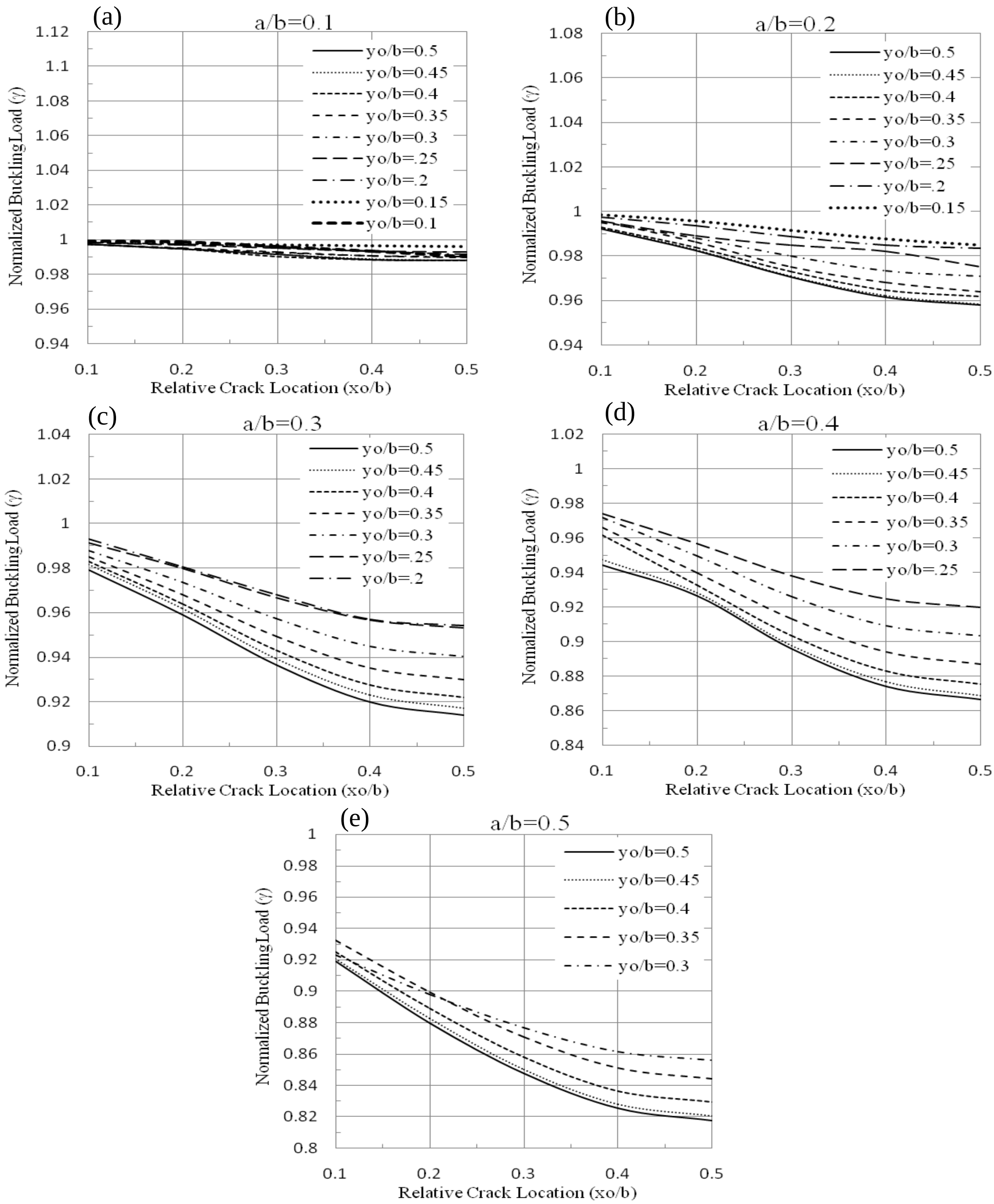



Figure (12) Variation of Normalized Buckling Load(γ) With Relative crack Location in x-direction(xo/b) For Different Relative Crack Locations in y-direction(yo/b) Under Loading Case(2)

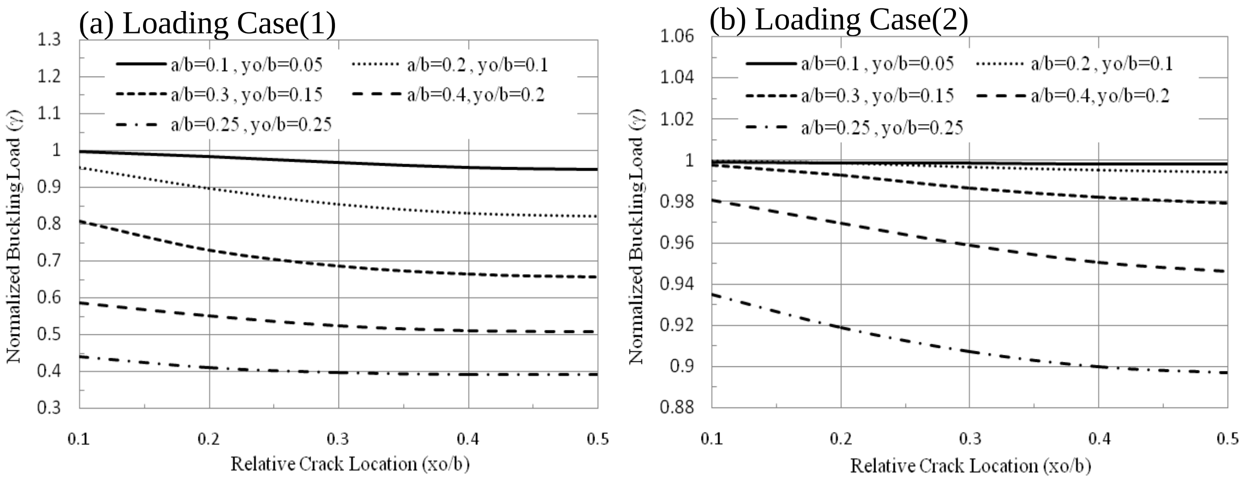



Figure (13) Variation of Normalized Buckling Load(γ) With Relative crack Location (xo/b) For Different Relative Crack Length of Edge Cracked Plate

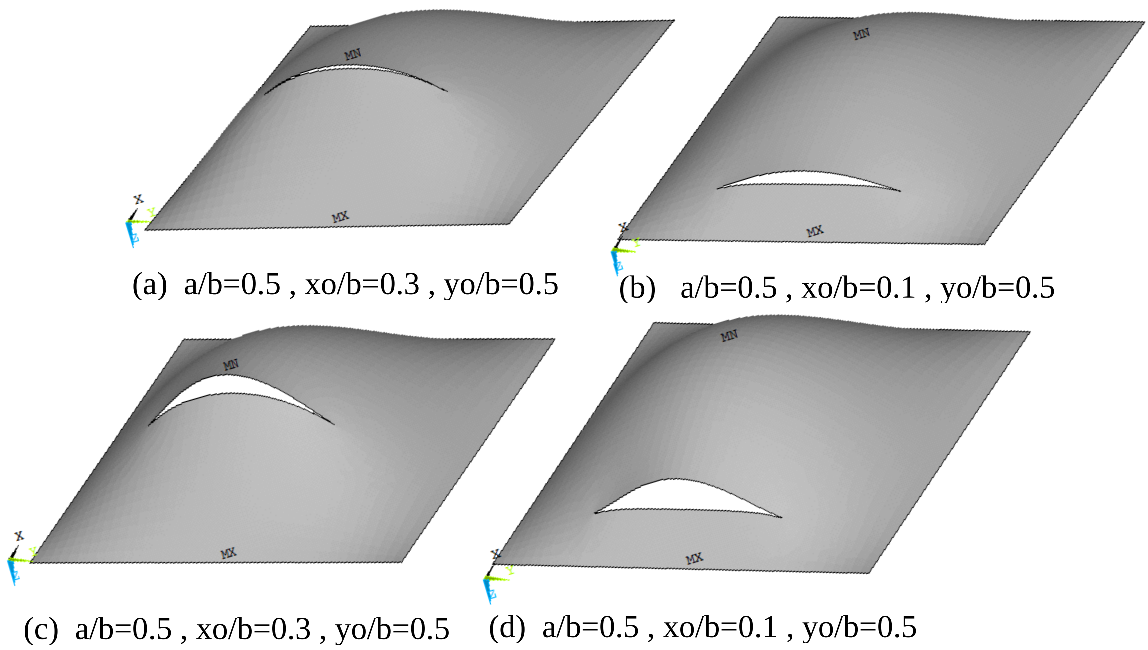



Figure (14) First Buckling Mode Shape of Cracked Plate:
(a,b) Loading Case(1) , (c,d) Loading Case(2)

## 5- Conclusions

In the present paper, the buckling behavior of cracked square thin plates subjected to compression load perpendicular or parallel load with respect to the crack faces was investigated. The influence of several crack parameters (i.e. crack length and crack location) on the buckling behavior was performed by utilizing the finite element method (ANSYS Software) in order to estimate the buckling load. Based on the results of this study, it can be concluded that :

(1) Under both loading situations, when the relative crack length is $a/b \geq 0.2$, a significant effect in the normalized buckling load (i.e. buckling strength) is expected. Small cracks $0.1 \leq a/b < 0.2$ may be ignored.

(2) The presence of a crack perpendicular to the loading direction in a plate under compression when $yo/b > 0.3$ and for all values of xo/b has a beneficial effect since it increases the buckling strength with respect to the uncracked case. While, reduction of the buckling strength is magnified with the crack parallel to the loading direction, which can be considered to be of "damaging type" on the buckling behavior.

(3) For edge cracked plates, the buckling strength more significantly decreases under both loading situations, particularly under perpendicular load with respect to crack faces, since it has the lowest buckling load. So can be considered to be the most dangerous one.

## References

[1] Sih G.C., Lee Y.D. Tensile and compressive buckling of plates weakened by cracks. Theoretical and Applied Fracture Mechanics 1986;6;129-138.

[2] Riks E., Rankin C.C., Brogan F.A. The buckling behavior of a central crack in a plate under tension. Engineering Fracture Mechanics 1992;43;529-548.

[3] Barut A., Madenci E. Buckling of a thin tension loading composite plate with an inclined crack. Engineering Fracture Mechanics 1997;58(3);233-248.

[4] Guz A.N., Dyshel M.Sh. Fracture and buckling of thin panels with edge crack in tension. Theoretical and Applied Fracture Mechanics 2001;36;57-60.

[5] Guz A.N., Dyshel M.Sh. Buckling and cracking characteristics of two-layered plate in tension. Theoretical and Applied Fracture Mechanics 2002;38;103-107.

[6] Vafai A., Estekanchi H.E. A parametric finite element study of cracked plates and shells. Thin-Walled Structures 1999;33;211-229.

[7] Estekanchi H.E, Vafai A. On the buckling of cylindrical shells with through cracks under axial load. Thin-Walled Structures 1999;35;255-274.

[8] Vafai A., Javidruzi M., Estekanchi H.E. Parametric instability of edge cracked plates. Thin-Walled Structures 2002; 40 ;29-44.

[9] Vafai A., Estekanchi H.E. Buckling of cracked cylindrical thin shells under combined internal pressure and axial compression. Thin-Walled Structures 2006;44;141-151.

[10] Kumar Satish Y.V., Paik J. K. Buckling analysis of cracked plates using hierarchical trigonometric functions. Thin-WalledStructures2004;42;687-700.

[11] Brighenti R. Numerical buckling analysis of compressed or tensioned cracked thin plates. Thin-Walled Structures 2005;27;265-276.

[12] Brighenti R. Buckling of cracked thin plates under tension or compression. Thin-Walled Structures 2005;43;209-224.

[13] Alinia M.M., Hosseinzadeh S.A.A., Habashi H.R. Influence of central cracks on buckling and post-buckling behavior

of shear panels. Thin-Walled Structures 2007;45;422-431.

[14] M.M., Hosseinzadeh S.A.A., Habashi H.R. Numerical modeling for buckling analysis of cracked shear panels. Thin-Walled Structures 2007;45;1058-1067.

[15] Paik J.K., Kumar Satish Y.V., Lee J. Myung. Ultimate strength of cracked plate elements under axial compression or tension. Thin-Walled Structures 2005;43;237-272.

[16] Paik J.K. Residual ultimate strength of steel plates with longitudinal cracks under axial compression-experiments. Ocean Engineering 2008;35;1775-1783.

[17] Paik J.K. Residual ultimate strength of steel plates with longitudinal cracks under axial compression-nonlinear finite element method investigations. Ocean Engineering 2009;36;266-276.

[18] Khedmati M.R., Edalat P., Javidruzi M. Sensitivity analysis of the elastic buckling of cracked plate elements under axial compression. Thin-Walled Structures 2009;47;522-536.

[19] Ventsel E and Krauthammer T. Thin plates and shells-theory, analysis and applications. New York 2001.